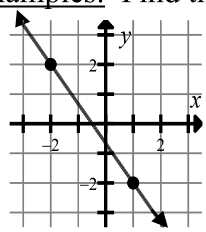
LESSON 1-1 SLOPES, LINES, CALCULATOR REVIEW

The slope of a line is symbolized by the letter "m".

Slope $= m = \dfrac{rise}{run} = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{\Delta y}{\Delta x}$

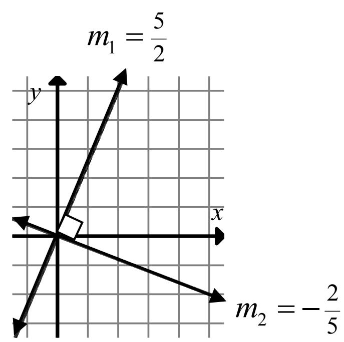
Examples: Find the slopes of the lines containing each pair of points.

1. 

2. $(-2,0)$ *and* $(4,2)$

3. $(3,2)$ *and* $(2,2)$

4. $(3,2)$ *and* $(3,5)$



Parallel lines have equal slopes ($m_1 = m_2$).
Perpendicular lines have slopes which are opposite reciprocals $\left(m_1 = -\dfrac{1}{m_2} \right)$.

Equations for lines

point-slope form: $y - y_1 = m(x - x_1)$
slope-intercept form: $y = mx + b$ (where b is the y-intercept)
general form : Ax + By + C = 0 (where A, B, and C are integers)

Examples: Find an equation of each line described.

5. a line through (2,3) with slope $m = -3$

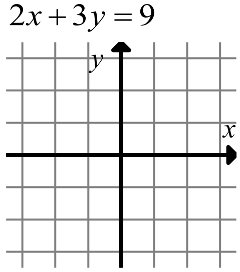
6. a vertical line through $(-1,2)$

7. a line through $(-1,2)$ parallel to the graph of $2x - 5y = 5$ (in slope-intercept form)

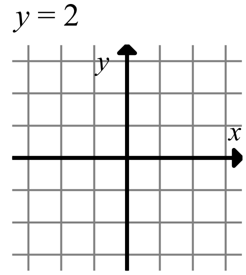
8. a line through $(-1,2)$ perpendicular to the graph of $2x - 5y = 5$ (in general form)

Examples: Draw a graph of each line.

9. $2x + 3y = 9$



10. $y = 2$



Calculator Examples:

11. Find a window to show a complete graph of $y = f(x) = -0.2x^3 - 2.2x^2 + 1.6x + 1$.
Indicate the scale on the graph or give your window setting.

12. Find the zeros of $y = f(x) = -0.2x^3 - 2.2x^2 + 1.6x + 1$.

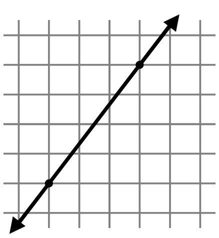
13. Find the points of intersection of $y = -x^3 + 12x^2 + 9x - 3$ and $3x - y + 5 = 0$. Write the equation you are solving.

14. Use a calculator to solve $\left|x^2 - 5\right| \geq 4$. Write your answer in both inequality notation and interval notation.

ASSIGNMENT 1-1

Find the slopes of these lines.

1.



2. through $(2,-6)$ *and* $(5,-12)$

3. through $(3,6)$ *and* $(-2,6)$

4. through $(-6,5)$ *and* $(4,3)$

Find an equation for each line.

5. through (1,2) with $m = -2$
6. through (2,0) and (3,1), in slope-intercept form
7. through (1,7) with undefined slope
8. through (1,7) with $m = 0$
9. vertical with x-intercept at 4
10. through $(-1,-3)$ parallel to the graph of $y = 3x - 5$, in general form
11. through (2,3) perpendicular to the graph of $2x - 3y = 7$
12. through $(2,-3)$ perpendicular to the graph of $x = 5$

Graph without using a calculator.

13. $y = -3x + 2$
14. $x = -2$
15. $2x + 5y + 10 = 0$

16. Show work to determine if $(3,5), (7,0)$, and $(-1,11)$ are collinear (lie on the same line).

Use a calculator for problems 17-27. Answers should be accurate to three or more decimal places (rounded or truncated).

17. Find an appropriate window to show a complete graph of $y = x^3 + 4x^2 - 5x$. Your window should show all zeros and all local maximum and minimum points (turn-around points). Draw a window rectangle on your own paper and accurately draw the graph. Indicate the scale on the graph or give the window setting.
18. Find the zeros of $y = f(x) = x^3 + 4x^2 - 5x$. Write the equation you are solving on your paper.
19. Copy and complete the table at the right for this same function from problem 18.

x	$f(x)$
7	
7.1	524.051
7.2	
7.3	
7.4	

Be sure to show three decimal places.

20. Find $f(-2.1576)$ for this same function.
21. Find the x- and y-coordinates of the local maximum and minimum points of $f(x)$.
22. Find the intersection points of the $f(x)$ function and $g(x) = -3x^2 - 5x + 15$. Write the equation you are solving.
23. Solve $x^3 + 4x^2 - 5x = -3x^2 - 5x$.
24. Solve $x^3 + 4x^2 - 5x \le 0$. Write your answer in <u>interval notation</u>. No work is required.
25. Find the points of intersection of the graphs of $x^2 + y = 4$ and $2x - y = 1$. Write the equation you are solving.
26. Find the <u>x-coordinate(s)</u> of the point(s) of intersection of the graphs of $x + y = 7$ and $2x - 3y = -1$. Write the equation you are solving.
27. Solve $\log(2x^2 - 5) = 0$.

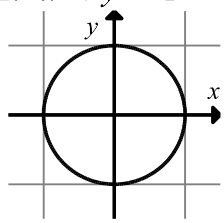
LESSON 1-2 FUNCTIONS, INVERSES, GRAPHING ADJUSTMENTS

Relation: any set of ordered pairs (any set of points on a graph)
Function: a special type of relation. y is a function of x if for each x-value there is only one y-value. The graph of a function passes the vertical line test. This is written $y = f(x)$.

$$\left.\begin{array}{ll}\text{Domain: } & \text{the set of all } x\text{-values} \\ \text{Range: } & \text{the set of all } y\text{-values}\end{array}\right\} \text{ assuming } y \text{ is a function of } x$$

Examples: Determine whether each is a function of x.

1. $x+y=1$
2. $x^2+y^2=1$



3. $y=-x^2+1$
4. $x+y^2=1$

Given: $f(x)=3x-1$ and $g(x)=x^2$. Find the following.

5. $f(10)=$
6. $g(x+\Delta x)=$
7. $g(f(x))=$
8. $(f \circ g)(x)=$

Determine the domain and range for each function.

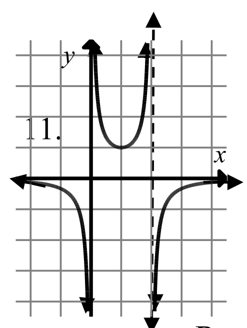
9. $f(x)=\sqrt{x-1}$

Do:
Ra:

10. $g(x)=\dfrac{1}{x-2}$

Do:
Ra:

11.



Do:
Ra:

One-to-one Function: a function in which not only is there only one y for each x, but there is also only one x for each y. The graph passes the horizontal line test as well as the vertical line test.

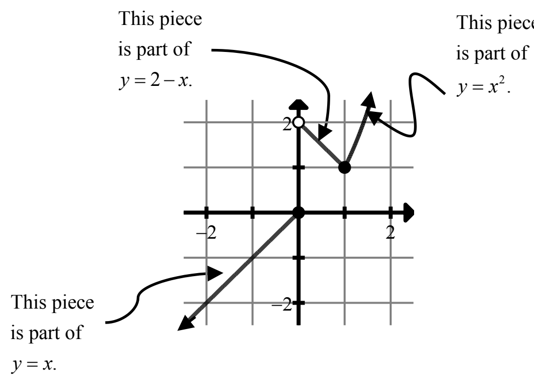
Inverse Function: found by switching x and y and solving for the new y. $f^{-1}(x)$ is the symbol for the inverse of $f(x)$. Only one-to-one functions have inverse functions. Since x and y are switched to produce inverse functions, the domain of f is the range of f^{-1} and vice versa. If (a,b) is in the f function, then (b,a) is in the f^{-1} function.

Examples:
12. Which of the relations in Examples 1-4 is a function with an inverse function?

13. Find the inverse of $f(x) = 2x^3 - 1$.

Piecewise Function: a function defined differently on different pieces of its domain.

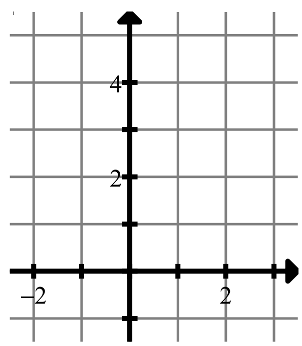
Example: $f(x) = \begin{cases} x, & x \le 0 \\ 2 - x, & 0 < x < 1 \\ x^2, & x \ge 1 \end{cases}$



Examples:
14. Graph this piecewise function and give the domain and range.

$f(x) = \begin{cases} |x|, & x < 1 \\ x + 2, & x \ge 1 \end{cases}$

Do:
Ra:



Zeros: x-values for which y equals zero.
Conventionally, zeros are written as single values (e.g. $x = 2$ or $x = 5$) while x-intercepts are written as ordered pairs (e.g. (2,0) or (5,0)).

Find the zeros without using a calculator.

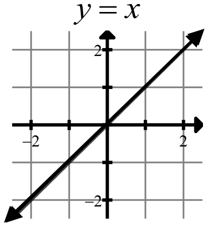
15. $f(x) = x^2 - 3x - 4$

16. $y = \dfrac{x^2 - 4}{x^2 + 4}$

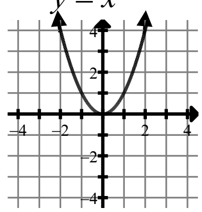
Parent Graphs

These graphs occur so frequently in this course that it would be worth your time to learn (memorize) them.

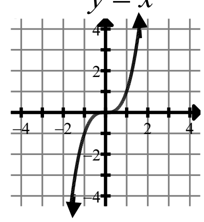
$y = x$



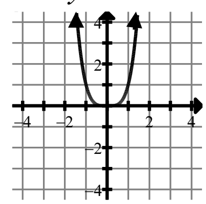
$y = x^2$



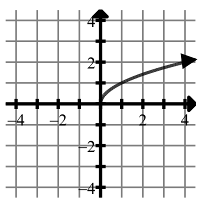
$y = x^3$



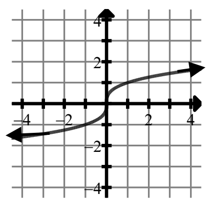
$y = x^4$



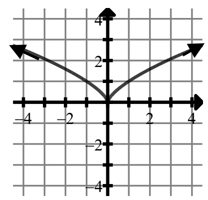
$y = \sqrt{x}$



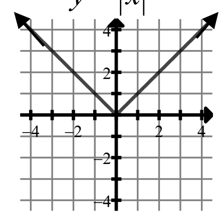
$y = \sqrt[3]{x}$



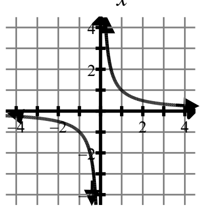
$y = x^{\frac{2}{3}}$



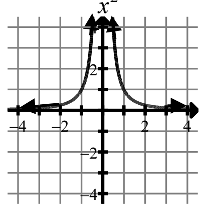
$y = |x|$



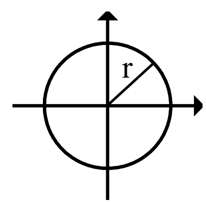
$y = \frac{1}{x}$



$y = \frac{1}{x^2}$



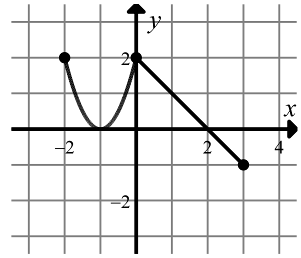
$x^2 + y^2 = r^2$



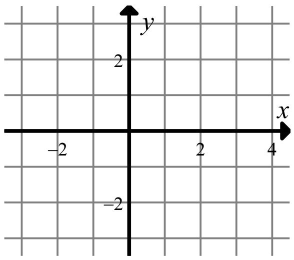
Graphing Adjustments to $y = f(x)$

1. $y = -f(x)$ reflect across the x-axis
2. $y = f(-x)$ reflect across the y-axis
3. $y = f(x) + d$ shift up if $d > 0$, shift down if $d < 0$
4. $y = f(x + c)$ shift left if $c > 0$, shift right if $c < 0$
5. $y = a \cdot f(x)$ vertical stretch if $a > 1$, vertical squeeze if $a < 1$
   (assumes a is positive, if a is negative a reflection is needed)
6. $y = f(b \cdot x)$ horizontal squeeze if $b > 1$, horizontal stretch if $b < 1$
   (assumes b is positive, if b is negative a reflection is needed)
7. $y = |f(x)|$ reflect all points below the x-axis across the x-axis. Leave points above the x-axis alone.
8. $y = f(|x|)$ eliminate completely all points left of the y-axis. Leave points right of the y-axis alone. Replace the left half of the graph with a reflection of the right half. Your graph should then show y-axis symmetry.

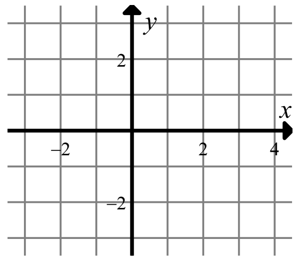
<u>Note:</u> Adjustments to functions always produce functions.
<u>Examples:</u> Use the graph of $y = f(x)$ shown to sketch the following:



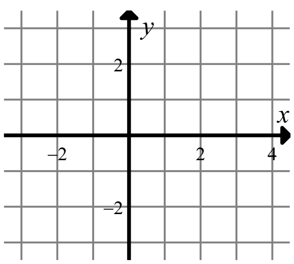
17. $y = f(x+2)$



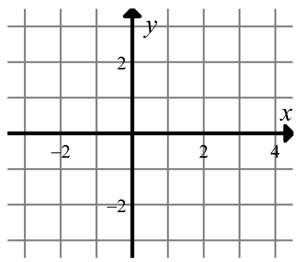
18. $y = -f(x)+2$



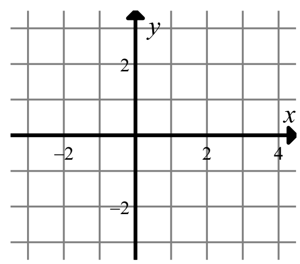
19. $y = \frac{1}{2} f(-x)$



20. $y = |f(2x)|$



21. $y = f(|x|)$



<u>ASSIGNMENT 1-2</u>

1. If $f(x) = 3x - 2$, find the following.
   a. $f(0)$ b. $f(-3)$ c. $f(b)$ d. $f(x-1)$

2. If $g(x) = \frac{|x|}{x}$, find the following.
   a. $g(2)$ b. $g(-2)$ c. $g(x^2)$

3. If $f(x)=x^2-x$, find $\dfrac{f(x+\Delta x)-f(x)}{\Delta x}$.

Without using a calculator, find the domain and range of the given function and draw its graph. When possible make use of the parent graphs in this lesson.

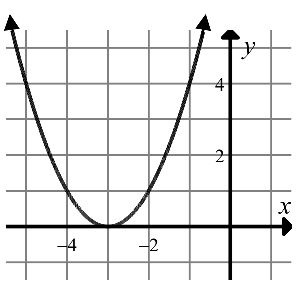
4. $f(x)=\sqrt{x+1}$ 5. $g(x)=x^2+2$ 6. $h(x)=4-x$

Without using a calculator determine whether y is a function of x.

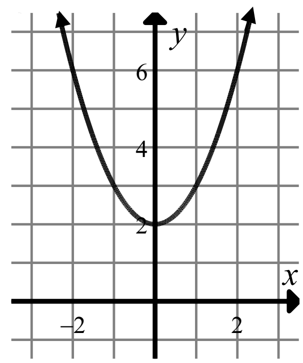
7. $2x+3y=4$ 8. $x^2+y^2=4$

9. Use the parent graph of $y=\sqrt{x}$ to graph the following.
   a. $y=\sqrt{x}+2$ b. $y=-\sqrt{x}$ c. $y=\sqrt{x-2}$ d. $y=2\sqrt{x}$

10. Use the parent graph of $y=x^2$ to determine an equation for each graph.
    a.



b.



11. If $f(x)=\sqrt{x}$ and $g(x)=x^2-1$, find the following.
    a. $f(g(1))$ b. $g(f(1))$ c. $(g\circ f)(x)$

12. If $f(x)=x+1$ and $g(x)=\dfrac{1}{x}$, find the following.
    a. $(f\circ g)(x)$ b. the domain of $(f\circ g)$
    c. $(g\circ f)(x)$ d. the domain of $(g\circ f)$

13. Are the two composite functions $(f\circ g)$ and $(g\circ f)$ from problem 12 equal?

14. If $f(x)=2x^2$, $g(x)=x+5$, $h(x)=2x-7$, and $k(x)=3$, find the following.
    a. $k(2)$ b. $f(k(x))$ c. $(f\circ f)(x)$
    d. $k(g(x))$ e. $(g\circ h)(3)$

Find the inverse function for each of the following showing organized work.

15. $y = 2x - 1$ 16. $f(x) = \sqrt[3]{x} - 1$ 17. $g(x) = x$ 18. $h(x) = \sqrt{x}$

19. Draw a graph of $h(x)$ and $h^{-1}(x)$ from problem 18. Did your answer on problem 18 include the domain restriction needed for $h^{-1}(x)$?

20. If $f(x) = \sqrt{x-2}$, $g(x) = x^2$, and $h(x) = \dfrac{1}{x^2}$, find the following.

a. $g(f(x))$ b. the domain of $(g \circ f)$

c. $h(f(x))$ d. the domain of $(h \circ f)$

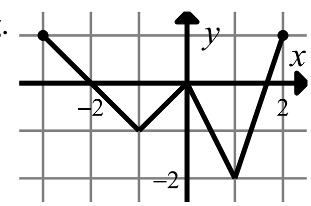
21. Without using a calculator graph this piecewise function.

$$f(x) = \begin{cases} x+2, & x < -2 \\ -x, & -2 \le x \le 2 \\ x^2 - 6, & x > 2 \end{cases}$$

Find the zeros of these functions without using a calculator.

22. $f(x) = \dfrac{x^2 - 3x + 2}{x^2 - 1}$ 23. $g(x) = 2x^3 - 8x$

Use the graph of $y = f(x)$ to draw **accurate** graphs of the following.



24. $y = -f(x)$ 25. $y = f(-x)$ 26. $y = |f(x)|$

27. $y = f(|x|)$ 28. $y = f(x) - 1$ 29. $y = f(x-1)$

30. $y = \frac{1}{2} f(x)$ 31. $y = f\left(\frac{1}{2}x\right)$ 32. $y = |f(x) - 1|$

Use a calculator for the rest of the assignment. Write answers to three or more decimal place accuracy.

33. Find the zeros of $f(x) = x^3 - 3x^2 - 2x + 4$.

34. Solve $x^3 - 2x^2 + 5 = \sqrt{3x+10}$.

Find the domain and the range for each function.

35. $y = 2x^2 + 3x + 6$ 36. $y = \dfrac{|x-2|}{x-2}$ 37. $y = \sqrt{7 - x^2}$

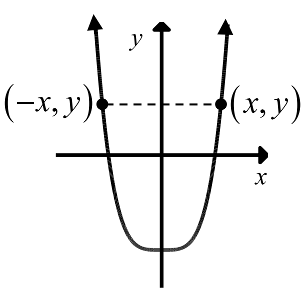
LESSON 1-3 INTERCEPTS, SYMMETRY, EVEN/ODD, INTERSECTIONS

x- and y-intercepts

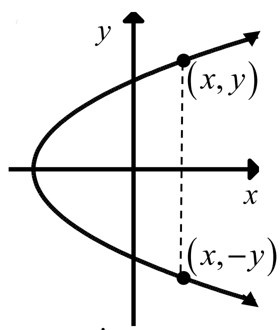
x-intercepts are points where a graph crosses or touches the x-axis. The y-coordinate is zero. To find the x-intercept, let $y = 0$ and solve for x.
y-intercepts are points where a graph crosses or touches the y-axis. The x-coordinate is zero. To find the y-intercept, let $x = 0$ and solve for y.

Example 1.
Find the x- and y-intercepts for $y^2 - 3 = x$.

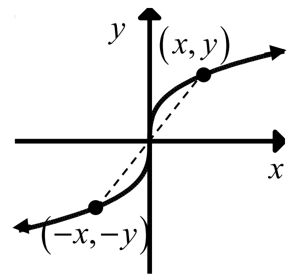
Symmetry



y-axis symmetry
reflection across the y-axis



x-axis symmetry
reflection across the x-axis



origin symmetry
reflection through the origin (0,0)

Graphs can be symmetric to other lines and points. However, we will concentrate on these three.

Formal tests for symmetry:
1. y-axis: replacing x with $-x$ produces an equivalent equation
2. x-axis: replacing y with $-y$ produces an equivalent equation
3. origin: replacing x with $-x$ and y with $-y$ produces an equivalent equation

Informal tests for symmetry:
1. y-axis: substituting a number and its opposite for x give the same y-value
2. x-axis: substituting a number and its opposite for y give the same x-value
3. origin: substituting a number and its opposite for x give opposite y-values

Note: These informal tests are not foolproof. Think about whether other numbers would work the same. If your substitution produces zero, try another number.

<u>Examples:</u> Find the type(s) of symmetry for the graph of:

2. $y = 2x^3 - x$ 3. $y = |x| - 2$ 4. $|y| = x - 2$

<u>Even/Odd Functions</u>

A function is defined to be **<u>even</u>** if $f(-x) = f(x)$ for all x in the domain of $f(x)$. Even functions have graphs with <u>y-axis symmetry</u>. Examples: $y = x^2$, $y = x^4$, $y = x^2 + 3$, $y = x^4 + x^2$

A function is defined to be **<u>odd</u>** if $f(-x) = -f(x)$ for all x in the domain of $f(x)$. Odd functions have graphs with <u>origin symmetry</u>. Examples: $y = x$, $y = x^3$, $y = x^5$, $y = x^5 - x^3$

<u>Examples:</u> Determine whether the following functions are even, odd, or neither.

5. $f(x) = x^3 - x$ 6. $g(x) = x^2 - 4$ 7. $h(x) = x^2 + 2x + 2$

<u>Points of Intersection of Two Graphs</u> <u>(without a calculator)</u>

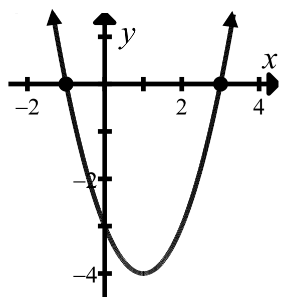
Method 1. Solve one equation for one variable and substitute into the other equation.

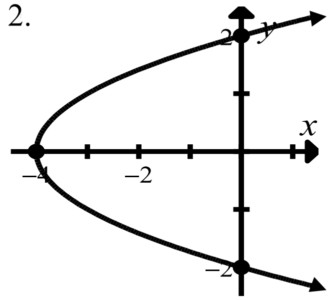
Method 2. Solve both equations for the same variable and set equal.

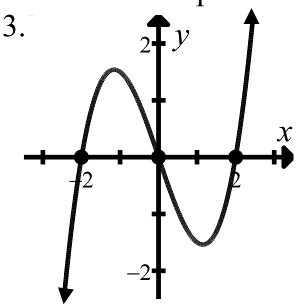
<u>Example 8.</u> Without using a calculator, find all points of intersection for the graphs of $x - y = 1$ and $x^2 - y = 3$.

ASSIGNMENT 1-3

Find the x- and y-intercepts for these graphs. Write your answers as ordered pairs.

1. 

2. 

3. 

Find the intercepts for the graphs of these equations. Do not use a calculator.

4. $y = 3x - 2$
5. $y = x^2 - 4x + 3$
6. $y = x\sqrt{x^2 - 9}$
7. $y = \dfrac{x-2}{x+3}$
8. $xy^2 + x^2 + 4y - 4 = 0$
9. $y = \sqrt{x^2 - 9}$

Check for x-axis, y-axis, or origin symmetry. Do not use a calculator.

10. the graph of Problem 1 on this assignment.
11. the graph of Problem 2 on this assignment.
12. the graph of Problem 3 on this assignment.
13. $y = x^2 - 2$
14. $y = x^3 + x$
15. $y = \dfrac{x}{x^2 + 1}$
16. $y^2 = x - 2$
17. $y = x^3 + 3$

18. Which of the graphs in Problems 1-3 represent(s) an odd function?
19. Which of the graphs in Problems 1-3 represent(s) an even function?

Without using a calculator determine whether the following functions are even, odd, or neither.

20. $f(x) = 4 - x^2$
21. $g(x) = x(x^2 - 4)$
22. $h(x) = x^3 - 1$

For Problems 23-25 find intercepts, symmetry, and sketch a graph without using a calculator.

23. $y = x + 2$
24. $y = \dfrac{1}{x}$
25. $y = x^2 + 3$

Find the points of intersection for the graphs of these equations without using a calculator. **Show algebra steps!**

26. $\begin{cases} y = x^3 \\ y = x \end{cases}$
27. $\begin{cases} x^2 + y^2 = 25 \\ y - x = 1 \end{cases}$

28. Is the point (1,4) on the graph of $2x - 3y = 10$?

29. If $(2,-1)$ is a point on the graph of $y = kx^3$, find the value of k.

Use a calculator to determine whether the following functions are even, odd, or neither.

30. $f(x) = \sqrt{x^2 - x^4}$

31. $g(x) = |x^3 - x|$

32. $h(x) = \sin x^3$

33. $j(x) = \log x^3$

For Problems 34-36 (using a calculator):
a. list the intercepts
b. identify the symmetry
c. draw a window rectangle and sketch the graph

34. $y = -2x^2 + 2x + 1$

35. $3x^2 + y^2 = 4$

36. $x + y^2 = 4$

Use a calculator on Problems 37-40. Answers should be accurate to three or more decimal places.

37. If $y = x^3 + 4x^2 - 5x$, find the value(s) of x when $y = 20$. Write the equation you are solving on your paper.
38. Solve $|3x - 7| < 9$.
39. Draw a window rectangle and sketch a graph of $y = (x-5)^{\frac{2}{3}}$.
40. Draw more than one window rectangle to show all local maximum and minimum points and end behavior of $f(x) = \frac{1}{4}x^4 - \frac{19}{6}x^3 - \frac{11}{4}x^2 + 5x$.

Find the slope for each of the following.

41. a line through (2,3) and (2,7)
42. the graph of $2x + 3y = 8$
43. a line perpendicular to $x = 4$

Find an equation for each of the following.

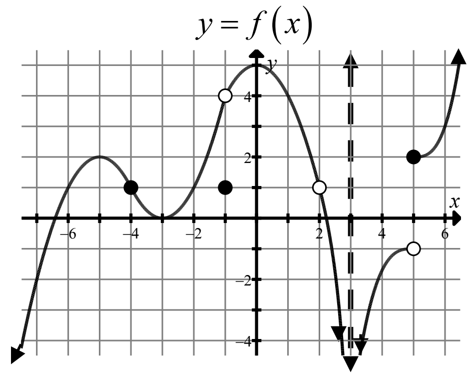
44. a line through (0,2) with slope $m = -\frac{2}{3}$
45. a line through (1,2) perpendicular to the graph of $y = \frac{2}{3}x$

LESSON 1-4 LIMITS , CONTINUITY

Limits

Informally, a **limit is a y-value** which a function approaches as x approaches some value.
$\lim_{x\to c} f(x) = L$ means as x approaches c, $f(x)$ approaches the y-value of L.

Examples



limits:

1. $\lim_{x\to -4} f(x) =$
2. $\lim_{x\to -1} f(x) =$
3. $\lim_{x\to 2} f(x) =$
4. $\lim_{x\to 3} f(x) =$
5. $\lim_{x\to 5} f(x) =$

one-sided limits:

6. $\lim_{x\to 5^-} f(x) =$
7. $\lim_{x\to 5^+} f(x) =$

function values:

8. $f(-4) =$
9. $f(-1) =$
10. $f(2) =$
11. $f(3) =$
12. $f(5) =$

Continuity

Informally, a function is continuous where it can be drawn without lifting a pencil.
Roughly, continuous means "connected."
Formally, a function is continuous where its limit and function value are the same.
In this course, we will work with three types of discontinuities: holes, vertical asymptotes, and jumps (breaks).

Example 13. List the x-values of the discontinuities of the function $y = f(x)$ graphed above.

All discontinuities can be classified as removable or nonremovable.
Removable discontinuities occur when the function has a limit (holes in the graph).
Nonremovable discontinuities occur when the limit of the function does not exist (jumps and vertical asymptotes).

Example 14. Which of the discontinuities from Example 13 are removable?

At x-values where a function is continuous, limits can be found by **direct substitution**.

Examples:

15. $\lim_{x\to 3} (3x^2 + 2) =$

16. $\lim_{x\to 1} \dfrac{x^2 + x}{x+1} =$

For **piecewise functions**, one-sided limit evaluation is often necessary.

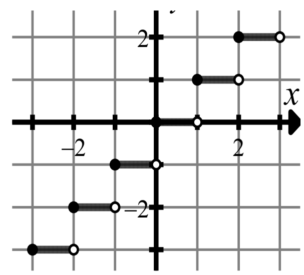
Examples:

17. If $f(x)=\begin{cases} 4-x, & x\le 1 \\ 4x-x^2, & x>1 \end{cases}$, $\lim\limits_{x\to 1} f(x)=$

18. If $g(x)=\begin{cases} 3x-x^3, & x\le 1 \\ 2x^2-1, & x>1 \end{cases}$, $\lim\limits_{x\to 1} g(x)=$

19. For this same g function, $\lim\limits_{x\to -1} g(x)=$

Another function requiring one-sided limit analysis is a step function called the **Greatest Integer Function** also known as the **Floor Function**. $f(x)=\lfloor x\rfloor=$ the greatest integer less than or equal to x. The graph is shown at the right.

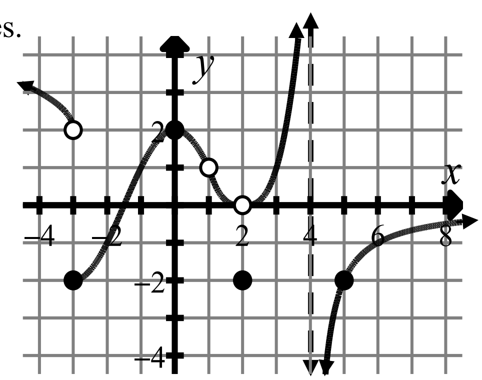


Examples: Find the following limits.

20. $\lim\limits_{x\to \frac{1}{2}} \lfloor x\rfloor=$ 21. $\lim\limits_{x\to 1} \lfloor x\rfloor=$

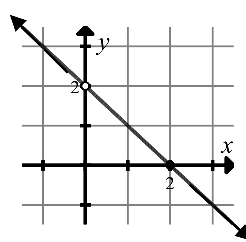
ASSIGNMENT 1-4

Use the graph of $y=f(x)$ at the right to find these values.



1. $\lim\limits_{x\to 5} f(x)$ 2. $\lim\limits_{x\to -3} f(x)$ 3. $\lim\limits_{x\to -3^-} f(x)$
4. $\lim\limits_{x\to -3^+} f(x)$ 5. $f(-3)$ 6. $\lim\limits_{x\to 4^-} f(x)$
7. $\lim\limits_{x\to 0} f(x)$ 8. $f(0)$ 9. $\lim\limits_{x\to 4} f(x)$
10. $\lim\limits_{x\to 4^+} f(x)$ 11. $f(4)$ 12. $f(2)$
13. $\lim\limits_{x\to 2} f(x)$ 14. $f(1)$ 15. $\lim\limits_{x\to 1} f(x)$
16. List the x-values of all removable discontinuities of $f(x)$.
17. List the x-values of all nonremovable discontinuities of $f(x)$.

Use the graph shown to find each value.



$$f(x)=\frac{-2x^2+4x}{2x}$$

18. a. $\lim\limits_{x\to 0} f(x)$ b. $\lim\limits_{x\to 2} f(x)$
c. $f(0)$ d. removable discontinuities

Use the graphs shown to find each value.

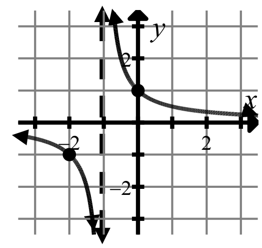
19. a. $\lim\limits_{x\to 0} g(x)$ b. $\lim\limits_{x\to -1^-} g(x)$ c. $\lim\limits_{x\to -1^+} g(x)$

d. $\lim\limits_{x\to -1} g(x)$ e. $g(-1)$ f. $g(0)$

g. $\lim\limits_{x\to -2} g(x)$ h. removable discontinuities

i. nonremovable discontinuities

$$g(x)=\frac{1}{x+1}$$



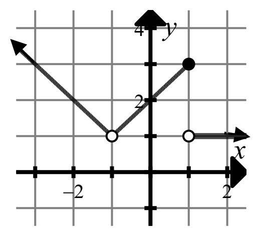
20. a. $h(-1)$ b. $h(1)$

c. $\lim\limits_{x\to -1} h(x)$ d. $\lim\limits_{x\to 1^-} h(x)$

e. $\lim\limits_{x\to 1^+} h(x)$ f. $\lim\limits_{x\to 1} h(x)$

g. removable discontinuities

h. nonremovable discontinuities



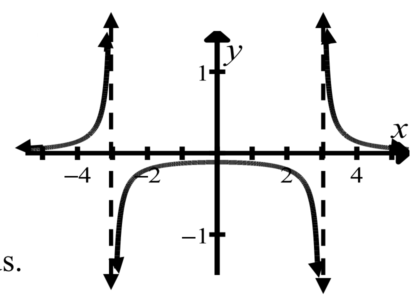
$$h(x)=\begin{cases} -x, & x<-1 \\ x+2, & -1<x\le 1 \\ 1, & x>1 \end{cases}$$

21. a. $\lim\limits_{x\to 3} f(x)$ b. $\lim\limits_{x\to 0} f(x)$

c. $\lim\limits_{x\to -3^-} f(x)$ d. $f(3)$

e. Use interval notation to show where f is continuous.

$$f(x)=\frac{1}{x^2-9}$$

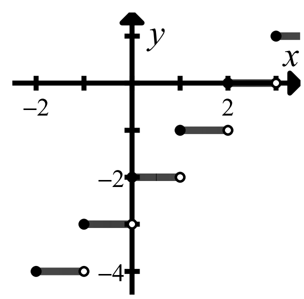


22. a. $g(1)$ b. $\lim\limits_{x\to 1} g(x)$

c. $\lim\limits_{x\to 1^-} g(x)$ d. $\lim\limits_{x\to 1^+} g(x)$

e. removable discontinuities

f. nonremovable discontinuities



$$g(x)=\lfloor x\rfloor-2$$

Find the indicated limits.

23. $\lim\limits_{x\to 2}(x^2-4)$ 24. $\lim\limits_{x\to -3}(x^2-x)$ 25. $\lim\limits_{x\to 0}\sqrt{x^2+9}$ 26. $\lim\limits_{x\to 3^-}(2x-5)$

27. $\lim\limits_{x\to -1}\dfrac{x}{x^2+1}$ 28. $\lim\limits_{x\to -2}\sqrt[3]{x^2+4}$ 29. $\lim\limits_{x\to 0}\dfrac{x}{x-1}$ 30. $\lim\limits_{x\to 0}(3x-3)^3$

Use these piecewise functions to find the following.

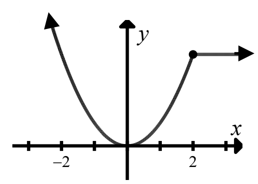
31. $f(x)=\begin{cases} x^2-2, & x\le 0 \\ x+2, & x>0 \end{cases}$ a. $\lim\limits_{x\to 0^-} f(x)$ b. $\lim\limits_{x\to 0^+} f(x)$ c. $\lim\limits_{x\to 0} f(x)$
d. Give the intervals where f is continuous.

32. $g(x)=\begin{cases} 3x^2-x, & x\le 2 \\ 2x+6, & x>2 \end{cases}$ a. $\lim\limits_{x\to 2} g(x)$ b. $\lim\limits_{x\to 0} g(x)$
c. List the interval(s) where g is continuous.

33. Find the interval(s) where $y=\sqrt{x-1}$ is continuous.

34. Find the discontinuities of $f(x)=\dfrac{x}{x(x+1)}$ and classify each of them as removable or nonremovable.

35. $f(x)=\begin{cases} x^2, & x\le 2 \\ a, & x>2 \end{cases}$



If $f(x)$ is a continuous function as shown in the graph, find the value of the unknown a.

36. Show algebraic steps (without a calculator) to find the intersection point(s) of the two functions $f(x)=\begin{cases} x^2+2, & x\le 1 \\ 3, & x>1 \end{cases}$ and $g(x)=-x+8$.

37. Verify your intersection point(s) from Problem 36 without using a calculator by graphing $f(x)$ and $g(x)$ on the same axes.

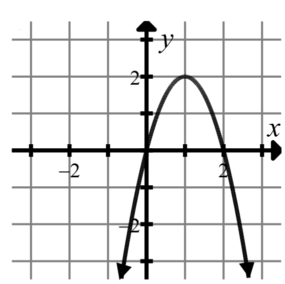
Use graphing adjustments to draw an accurate graph of each of the following without using a calculator. If necessary, use the parent graphs shown on Page 6.

38. $y=|x+2|$ 39. $y=|x|+2$ 40. $y=\dfrac{1}{x-1}$ 41. $y=\sqrt{|x|}$

42. $y=2\sqrt[3]{x}$ 43. $y=-\dfrac{1}{x^2}$ 44. $y=\dfrac{1}{(-x)^2}$ 45. $y=(x+3)^3-2$

46. Write an equation for the curve shown.

47. Check your answer by graphing with a calculator.



UNIT 1 SUMMARY

Slope: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

Equations for lines:

Point-Slope form $y - y_1 = m(x - x_1)$

Slope-Intercept form $y = mx + b$ (where b is the y-intercept)

Domain: all possible x-values
Range: all possible y-values

Inverse functions: found by switching x and y and solving for the new y.

Parent graphs and graphing adjustments: see Page 6
Intercepts:

To find the x-intercept, let $y = 0$ and solve for x.
To find the y-intercept, let $x = 0$ and solve for y.

Symmetry:
Informal tests:
1. y-axis: substituting a number and its opposite for x give the same y-value.
2. x-axis: substituting a number and its opposite for y give the same x-value.
3. origin: substituting a number and its opposite for x give opposite y-values.

Even/odd functions:
Even functions have graphs with y-axis symmetry.
Odd functions have graphs with origin symmetry.

Limits:
A limit is a y-value.
Analyze left and/or right behavior.
Use direct substitution.

Discontinuities: holes, vertical asymptotes, and jumps (breaks).
Removable (holes). Nonremovable (jumps and vertical asymptotes).

ASSIGNMENT 1-5 REVIEW

Draw accurate graphs for the following without using a calculator. Use the parent graphs on Page 6 to help you whenever possible.

1. $4x+2y=6$ 2. $y=\dfrac{-x+4}{2}$ 3. $y=\dfrac{1}{x}+1$ 4. $y=\sqrt[3]{x-2}$ 5. $y=\left|x^2-2\right|$

6. $y=x^{\frac{2}{3}}-1$ 7. $y-x^2=0$ 8. $x=y^2$ 9. $y=x^3-1$

10. For which of the relations in Problems 1-9 is y not a function of x?

11. Find equations for lines passing through $(-1,3)$ with the following characteristics.

a. $m=\frac{2}{3}$ b. parallel to $2x+4y=7$

c. passing through the origin d. perpendicular to the x-axis

Without using a calculator, find the point(s) of intersection of the graphs of the following. **Show algebra steps!**

12. $\begin{cases} y=x+5 \\ y=-2x+8 \end{cases}$ 13. $\begin{cases} x^2-y^2=9 \\ x^2+y^2=9 \end{cases}$

14. If $f(x)=1-x^2$ and $g(x)=2x+1$, find the following.

a. $f(x)+g(x)$ b. $f(g(x))$ c. $(g\circ f)(2)$

Find the zeros without using a calculator.

15. $f(x)=x^4-7x^2+12$ 16. $g(x)=\begin{cases} 2x-1, & x<0 \\ x^2-4, & x\geq 0 \end{cases}$

17. Find the inverse function for $f(x)=\left(x^3-1\right)^5$.

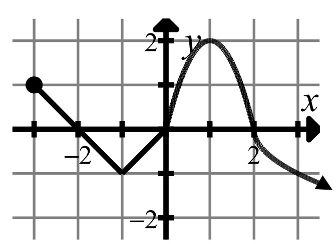
For each function in Problems 18-20, without using a calculator:
a. find the domain and the range.
b. find the intercepts.
c. discuss the symmetry.
d. tell whether the function is even, odd, or neither.
e. draw an accurate graph.

18. $y=\left|x^2-4\right|$ 19. $y=-\sqrt{x+4}$ 20. $y=\left|x^3\right|-2$

Use the graph of $y=f(x)$ at the right to draw an accurate graph for each of the following.

21. $y=\frac{1}{2}\left|f(x)\right|$ 22. $y=\left|f(2x)\right|$

23. $y=f(x-2)-2$



Use a calculator for Problems 24-29. Remember to show three or more decimal place accuracy for all answers that are not exact.

For Problems 24-26 :

a. find the domain and the range.
b. find the intercepts.
c. discuss the symmetry.
d. tell whether the function is even, odd, or neither.
e. draw an accurate graph.

24. $y = \frac{x}{x^2 - 4}$ 25. $y = -\sqrt{5 - 2x^2}$ 26. $y = 3x^2 - 3x - 5$

27. Solve $3x^3 - 3x + 1 \le 0$.

28. Solve $|3x + 5| > 2$.

29. Find the x-value(s) of the point(s) of intersection for the graphs of $x - y^2 = -7$ and $2x - 3y + 12 = 0$. Write the equation you are solving.

Are the following functions even, odd, or neither?

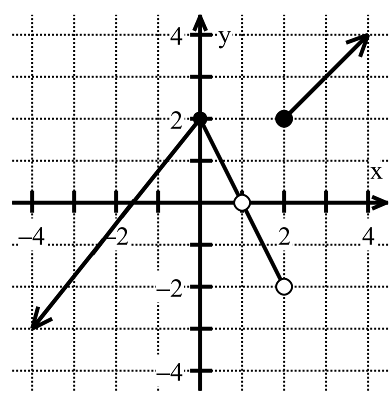
30. $g(x) = \frac{x}{|x^3 - x|}$ 31. $h(x) = \frac{x-1}{|x^3 - x|}$

Find the following limits without using a calculator.

32. $\lim_{x \to -2} (3x - 3)$ 33. $\lim_{x \to 2^-} \left\lfloor \frac{x}{2} - 4 \right\rfloor$ 34. $\lim_{x \to 2} \left\lfloor \frac{x}{2} - 4 \right\rfloor$ 35. $\lim_{x \to 3^+} \left\lfloor \frac{x}{2} - 4 \right\rfloor$

Use the graph of $y = f(x)$ for Problems 36-42.

Find the following limits and function values.



36. $\lim_{x \to 2} f(x)$ 37. $f(2)$ 38. $\lim_{x \to 2^-} f(x)$

39. $\lim_{x \to 1} f(x)$ 40. $\lim_{x \to 0} f(x)$

41. List all removable discontinuities of $f(x)$.

42. List all nonremovable discontinuities of $f(x)$.

Use the function $g(x) = \begin{cases} x - 1, & x \le 0 \\ x^2 - 1, & 0 < x < 2 \\ 4, & x \ge 2 \end{cases}$ for Problems 43-48.

43. Sketch a graph of $g(x)$.

Find the following limits.

44. $\lim_{x \to 0} g(x)$ 45. $\lim_{x \to 2} g(x)$ 46. $\lim_{x \to 2^-} g(x)$ 47. $\lim_{x \to 1} g(x)$

48. List all discontinuities of $g(x)$.